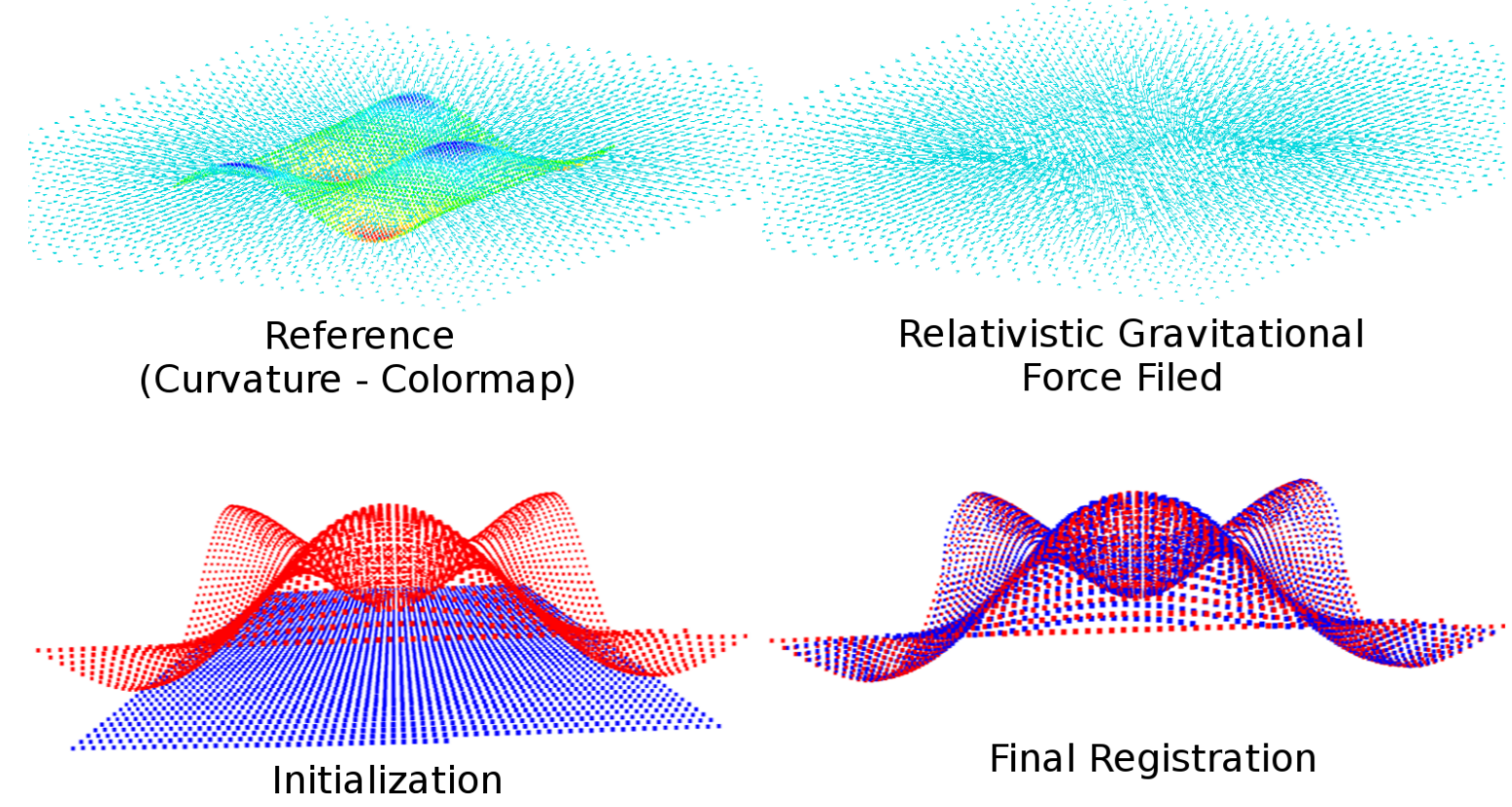


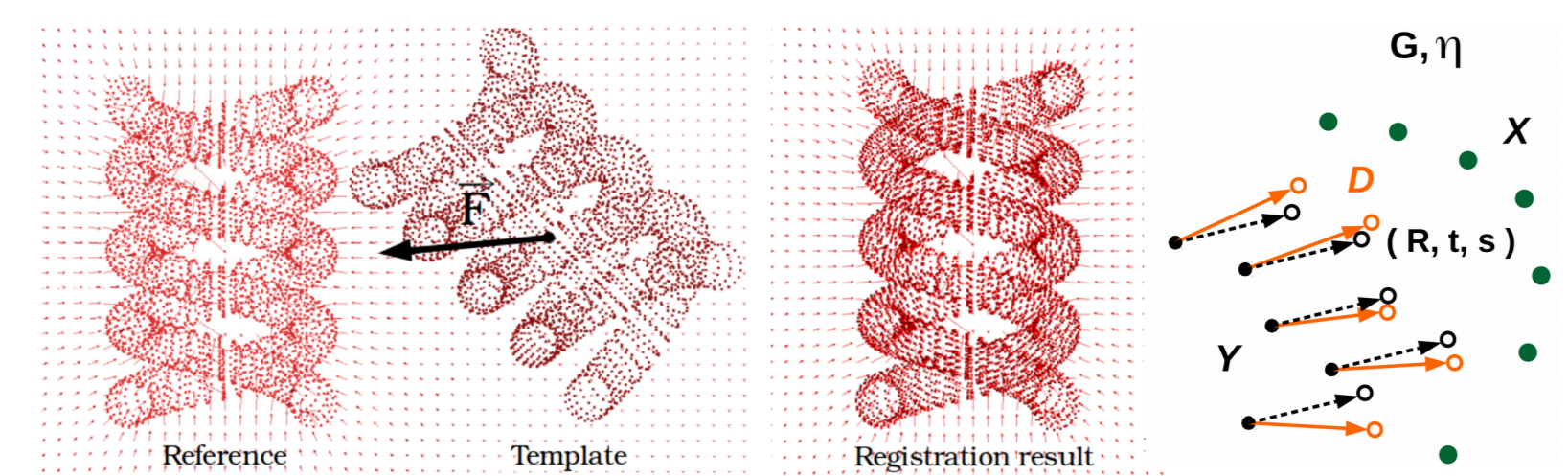
Overview

- NRGA** is physics-based non-rigid point set registration method which estimates **correspondences** and **transformations** between 'template' and 'reference' point sets
- Distributed** and **collision-less N-body** simulation is performed on several regions of point sets; the attractive gravitational force has relativistic effect; the **Coherent Collective Motion** operator [3] regularizes distributed position updates
- Motivation:** a parallelizable non-rigid point set registration method which is robust against missing input data and noise; better balances correspondence accuracy and geometric consistency

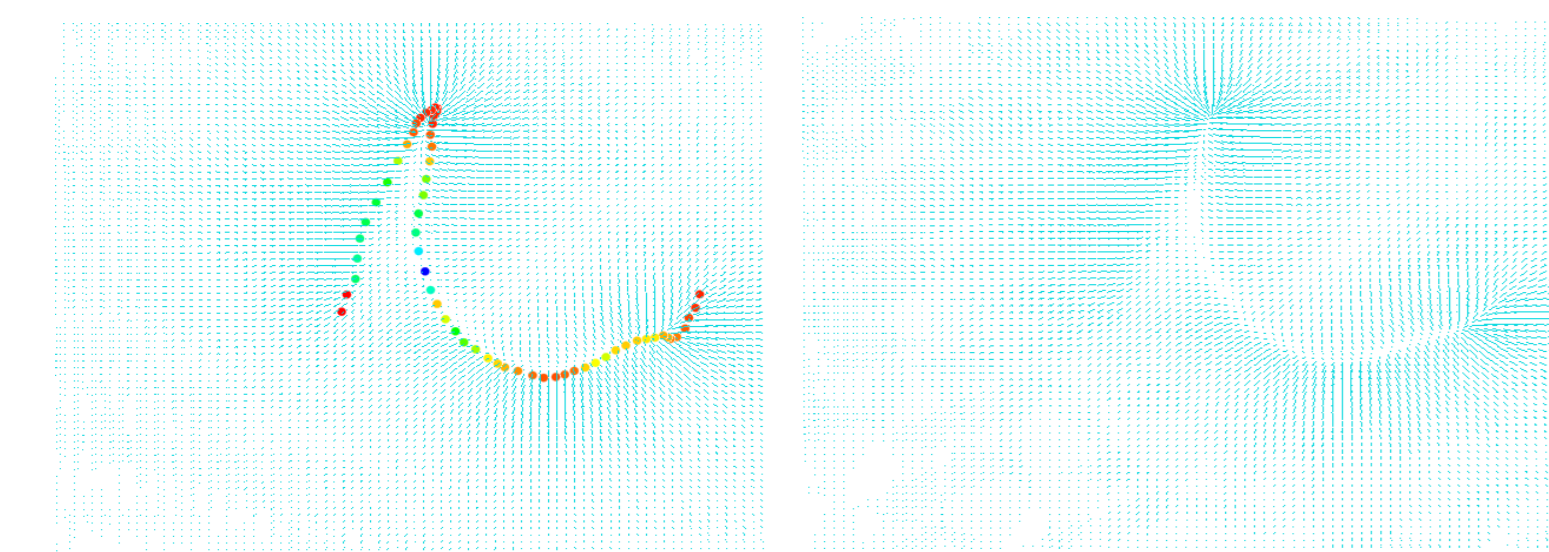


Related Work

NRGA is related to its rigid counterpart **GA** [1] which estimates a single global rigid transformation parameter using **Absolute Orientation** method or **Kabsch Algorithm** [4]



The gravitational force function used in [1] is altered in NRGA by relativistic effect as in [2]



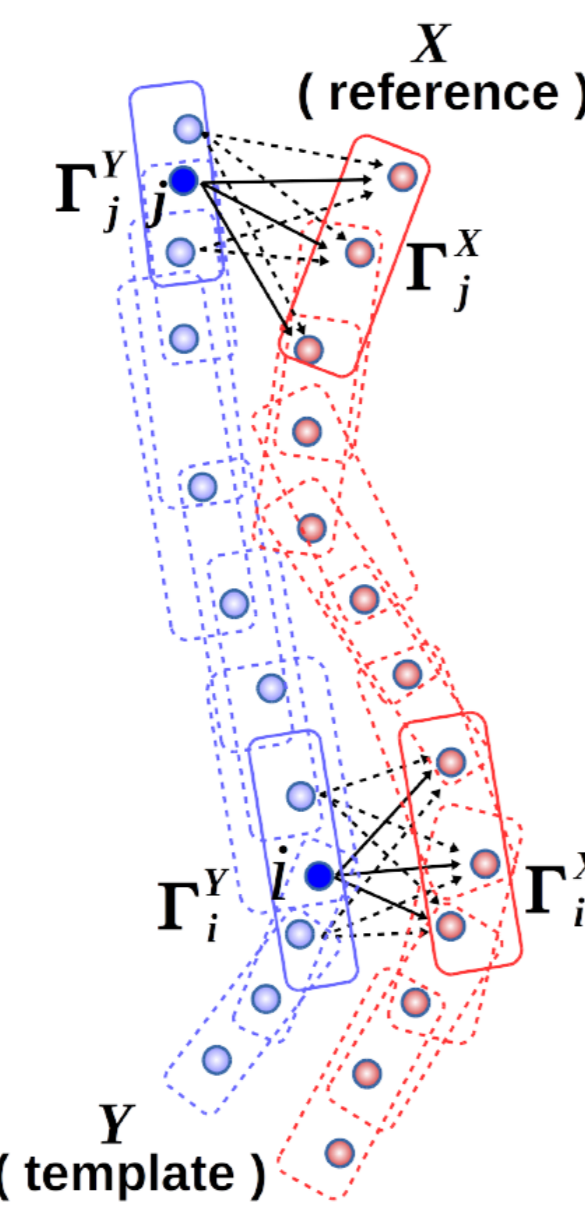
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Proposed NRGA

Distributed N-Body Simulation

Gravitational Potential Energy is the Distance Transform function $\Rightarrow \arg \min_{\mathbf{T}} \sum_{i=1}^M \sum_{j=1}^{|\mathcal{N}(i)|} \omega_{ij} (d(\mathcal{T}(\mathbf{Y}_i, \mathbf{T}_i) - \mathbf{X}_j))^{-1}$



$$\mathbf{F}_{p \in \Gamma_k^Y}^a = -G m_p \sum_{q \in \Gamma_k^X} \frac{m_q \left(\mathbf{r}_p \left(1 - \frac{\kappa_q r_{pq}^2}{2} \right) - \mathbf{r}_q \right)}{\left(\|\mathbf{r}_p - \mathbf{r}_q\|^2 + \epsilon^2 \right)^{\frac{3}{2}} \left(1 - \frac{\kappa_q r_{pq}^2}{4} \right)^{\frac{3}{2}}}$$

Relativistic Gravitation Force (RGF) field parameterized by per-point **Gaussian Curvature** of 'reference'

$$\mathcal{F}_k^t = [\dots, \mathbf{F}_p^a - \eta v_p^t, \dots]^T$$

$$\mathcal{V}_k^t = [\dots, v_p^t, \dots]$$

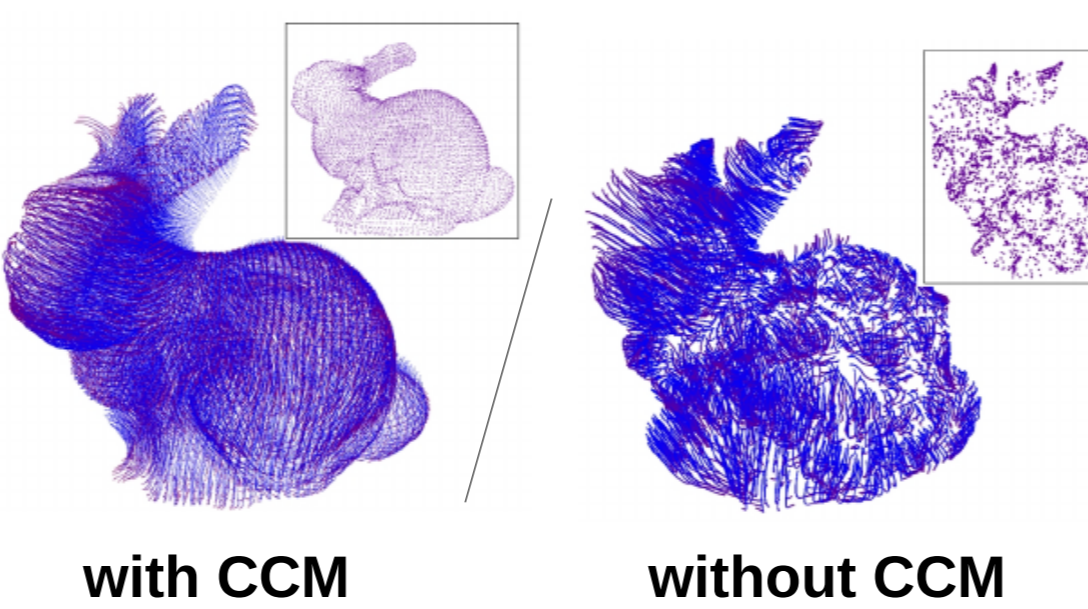
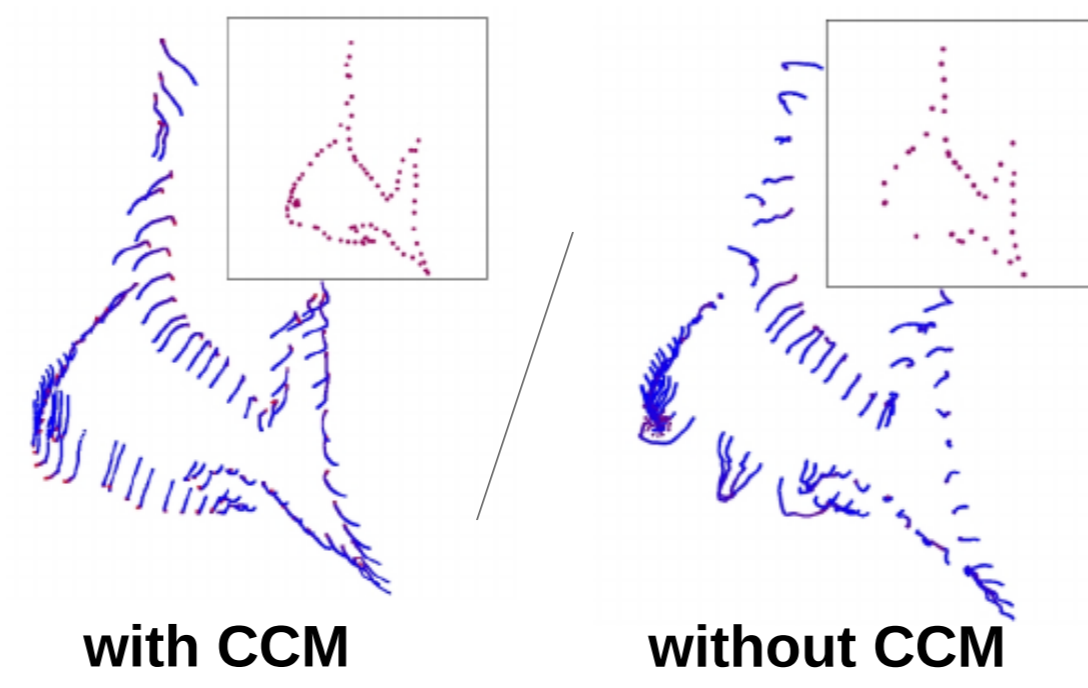
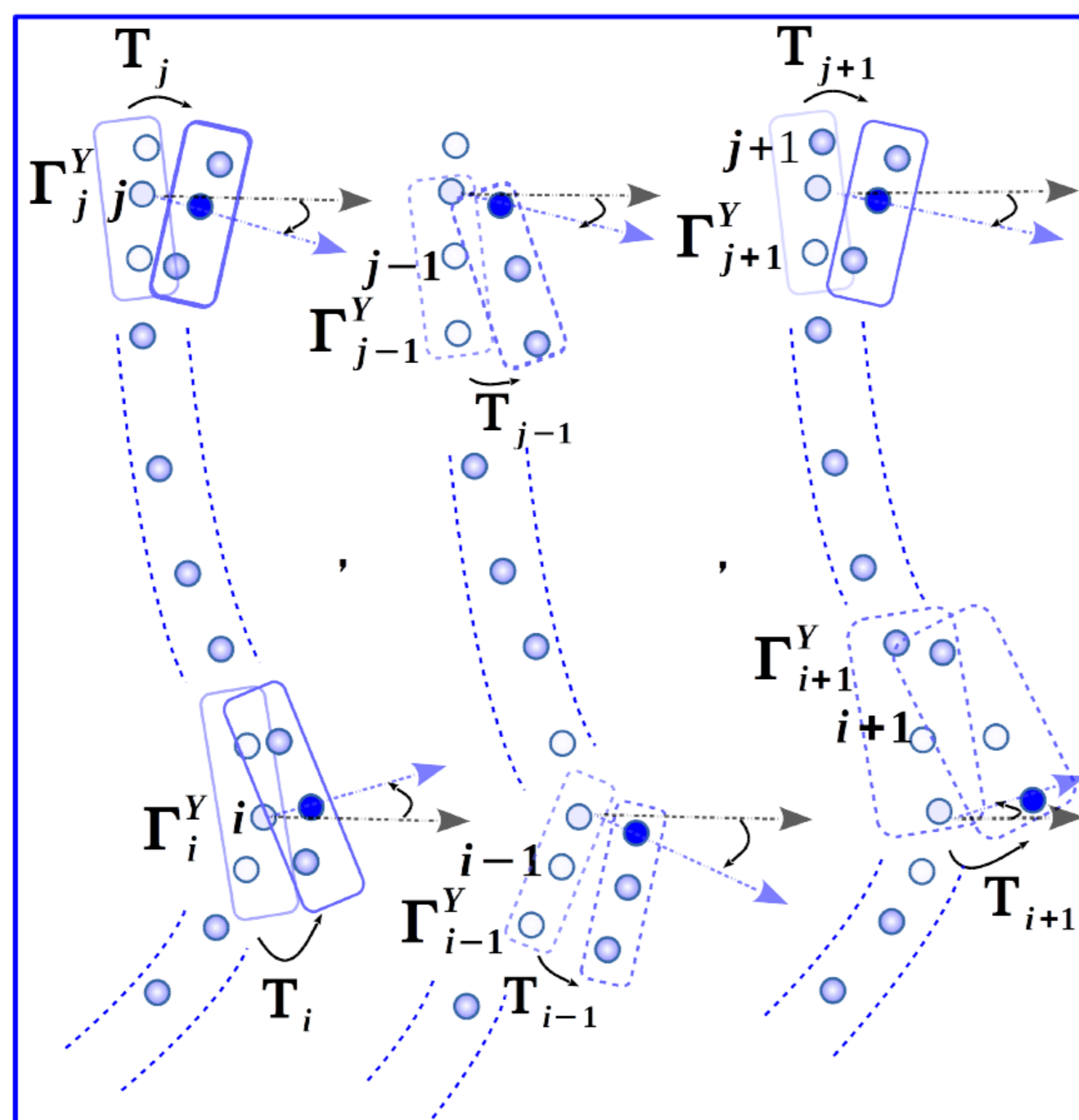
$$\mathcal{V}_k^{t+1} = \mathcal{V}_k^t + \Delta t \mathcal{F}_k^t \circ [\dots, m_p^{-1}, \dots]^T$$

$$\mathcal{D}_k^{t+1} = \Delta t \mathcal{V}_k^{t+1}$$

← Motion Update using Euler time integration

i) Locally Multiply-Linked N-Body Interaction

Coherent Collective Motion (CCM)



ii) Ξ (Coherent Collective Motion)

- The CCM is a locally-aware global topology preserving operator
- The nature of collective particle motion is similar to **Smoothed Particle Hydrodynamics**

$$\Xi(v_i^t) = |v_i^t| \left(\sum_{k \in \Psi_i} v_k^t \right)$$

← Gives directional Coherency

$$\Xi(\mathbf{T}_i) = \left\{ (\vartheta) \left(\sum_{k \in \Psi_i} \mathbf{R}_k \right), (\vartheta) \left(\sum_{k \in \Psi_i} \mathbf{t}_k \right), (\vartheta) \left(\sum_{k \in \Psi_i} \mathbf{s}_k \right) \right\}$$

← Gives Orientation Coherency

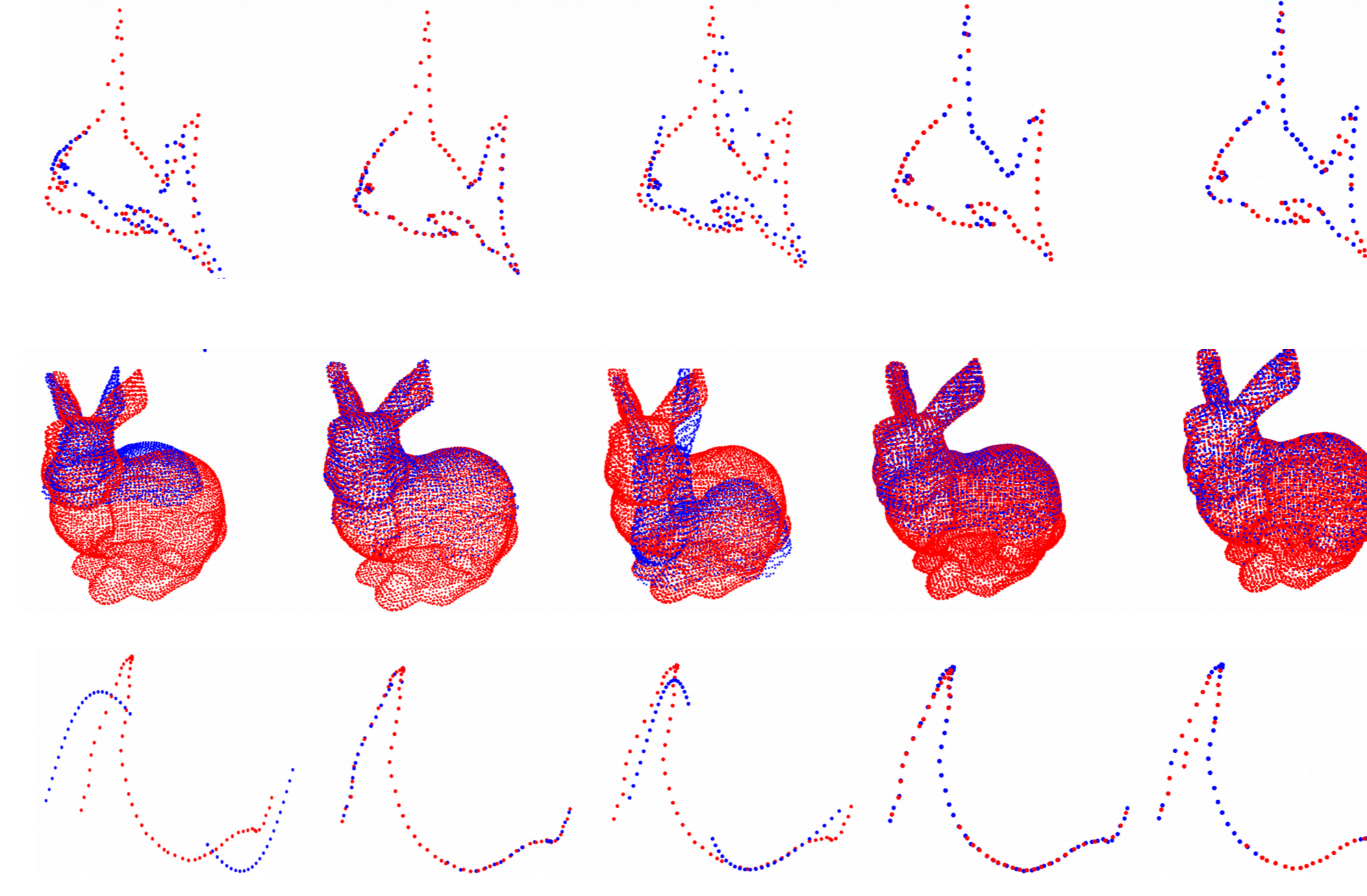
Complexity Analysis

$$\mathcal{O} \left(\underbrace{\xi}_{\text{iterations}} \left(\underbrace{M \log M}_{\text{k-d tree}} + \underbrace{M \rho M \rho N}_{\text{NRGA: N-body}} + \underbrace{M}_{\text{NRGA: CCM}} \right) \right) = \mathcal{O} \left(M^2 N \right)$$

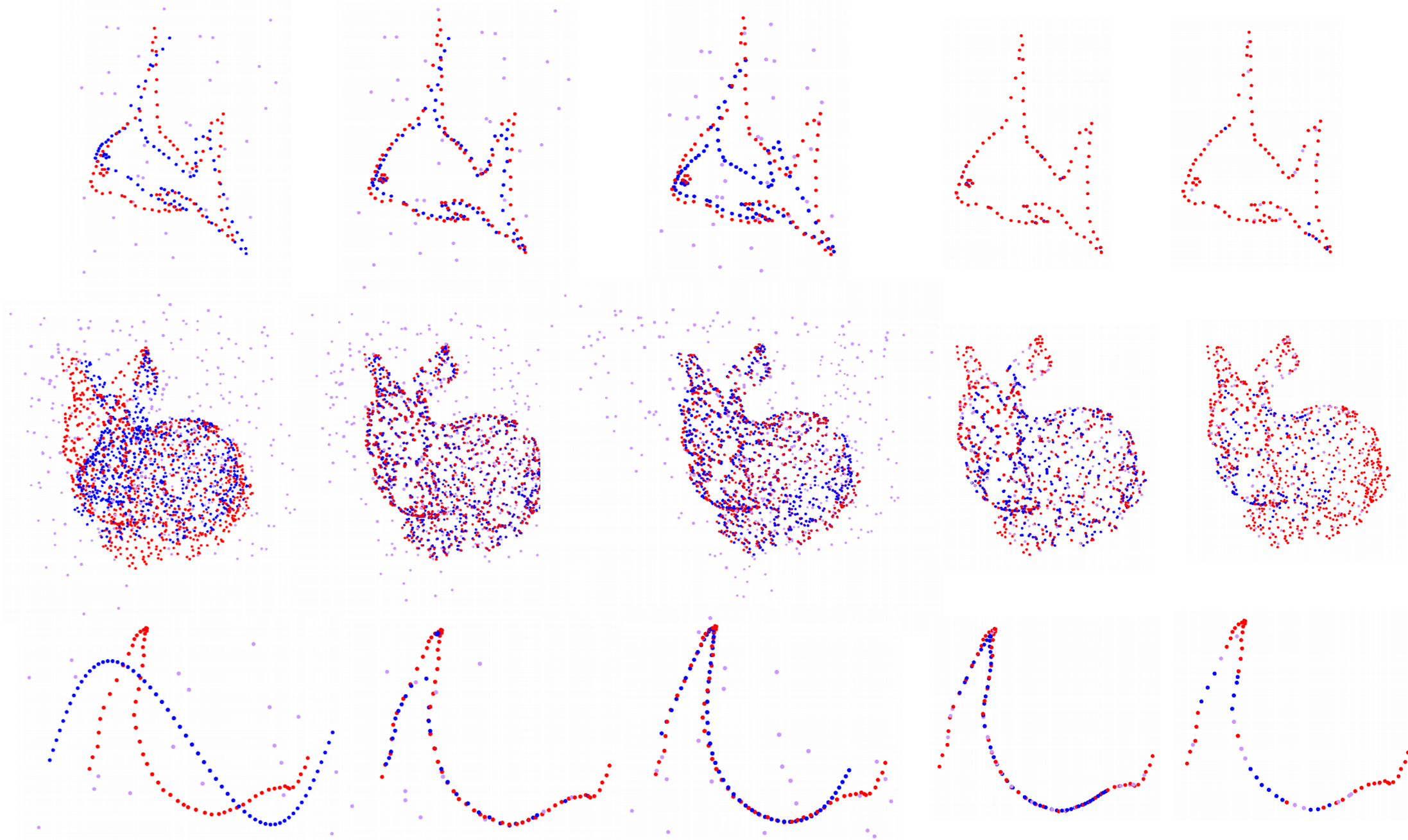
- Distributed N-body simulation on the regions can be solved using Fast Multi-Pole Method (FMM) or Barnes-Hut algorithm

Results

Missing Data (> 30%)



Uniform Noise (30%)



Expression Matching

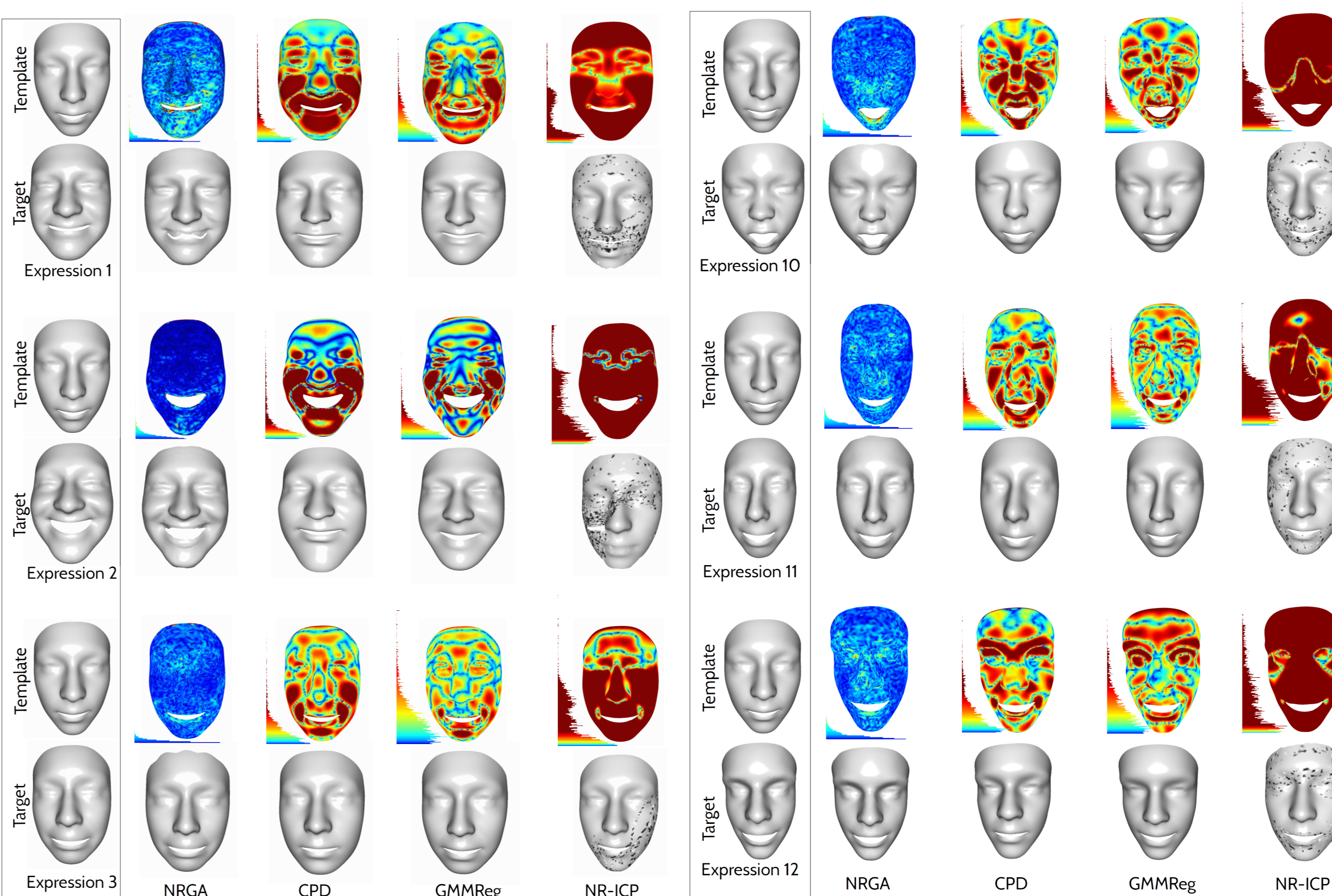
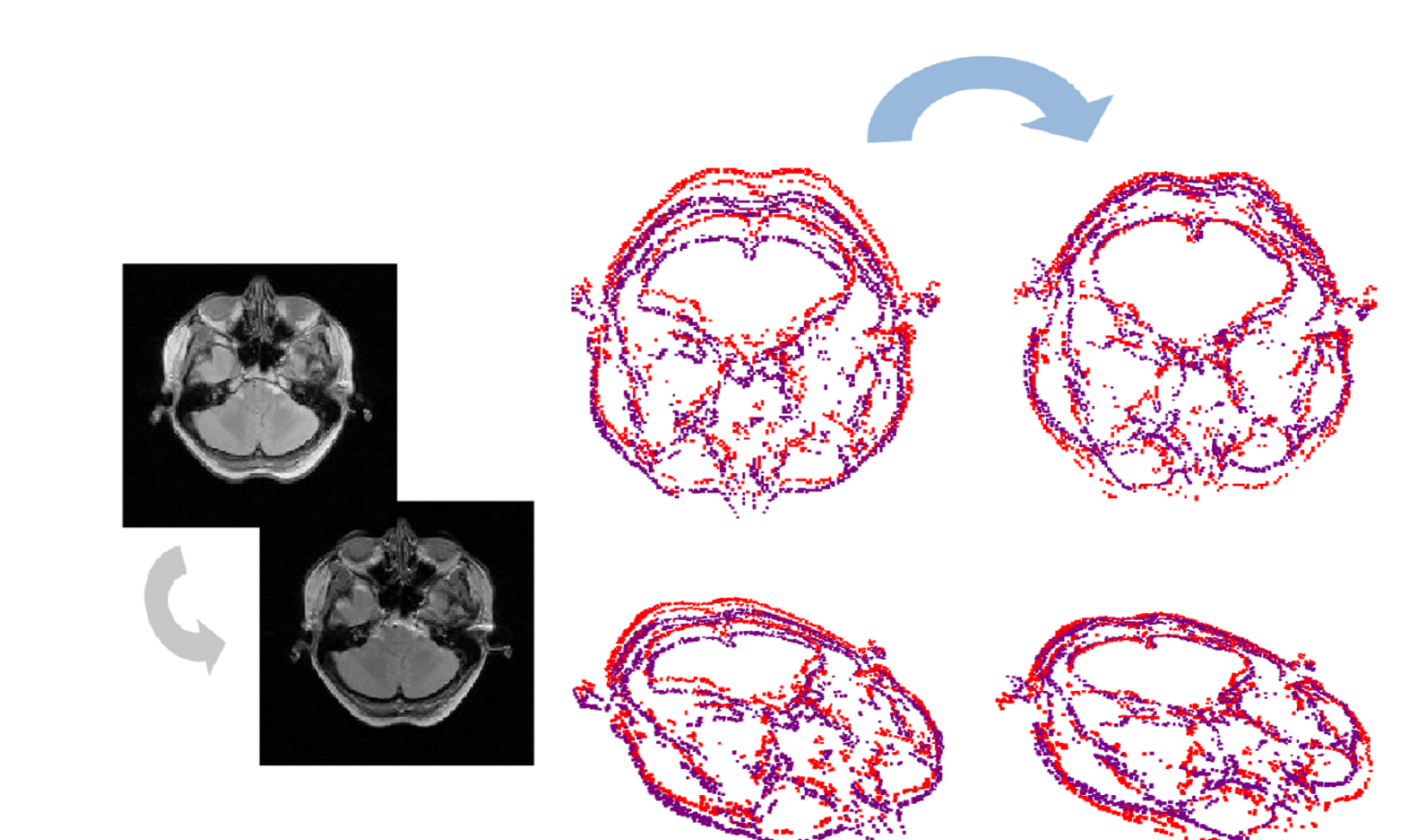
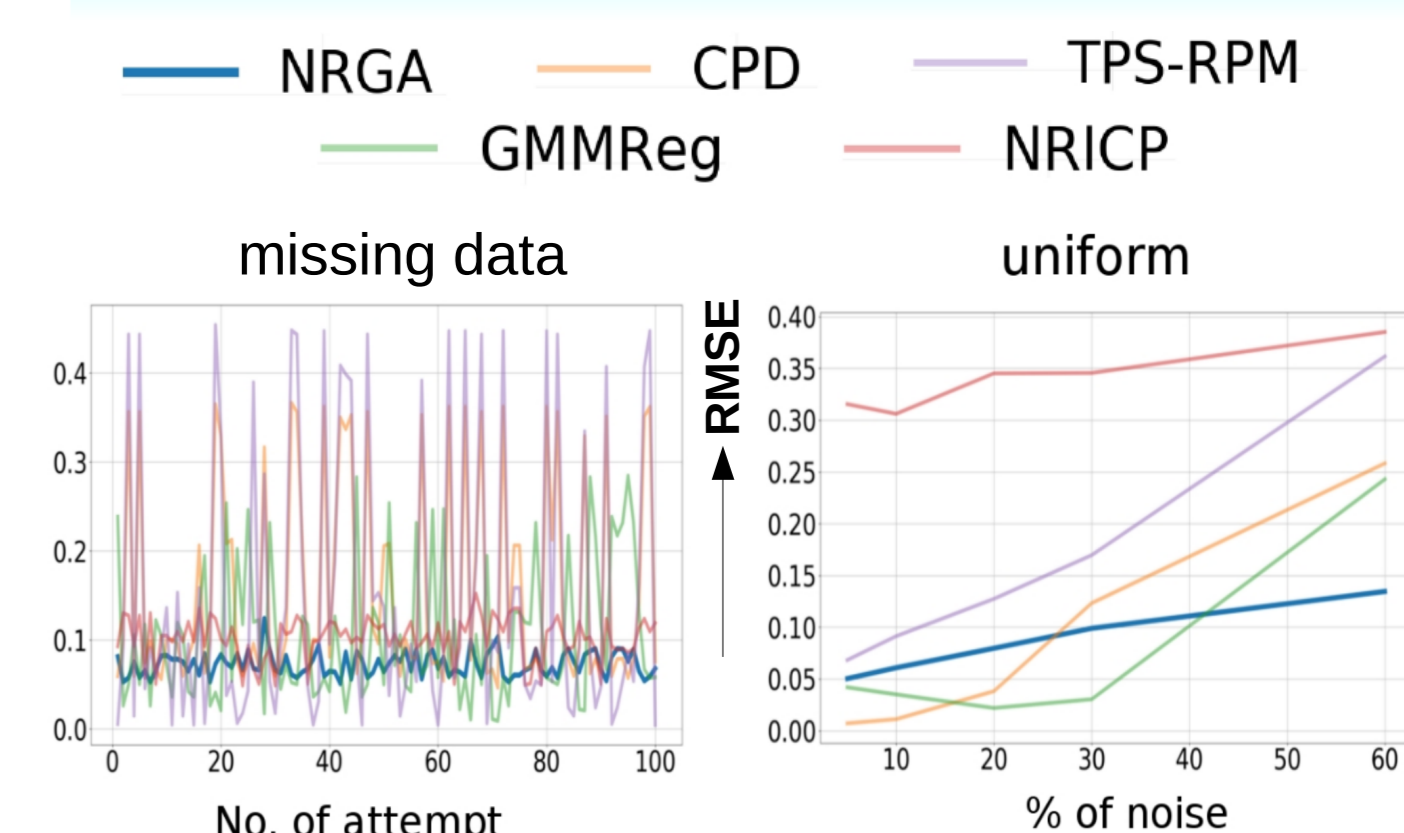


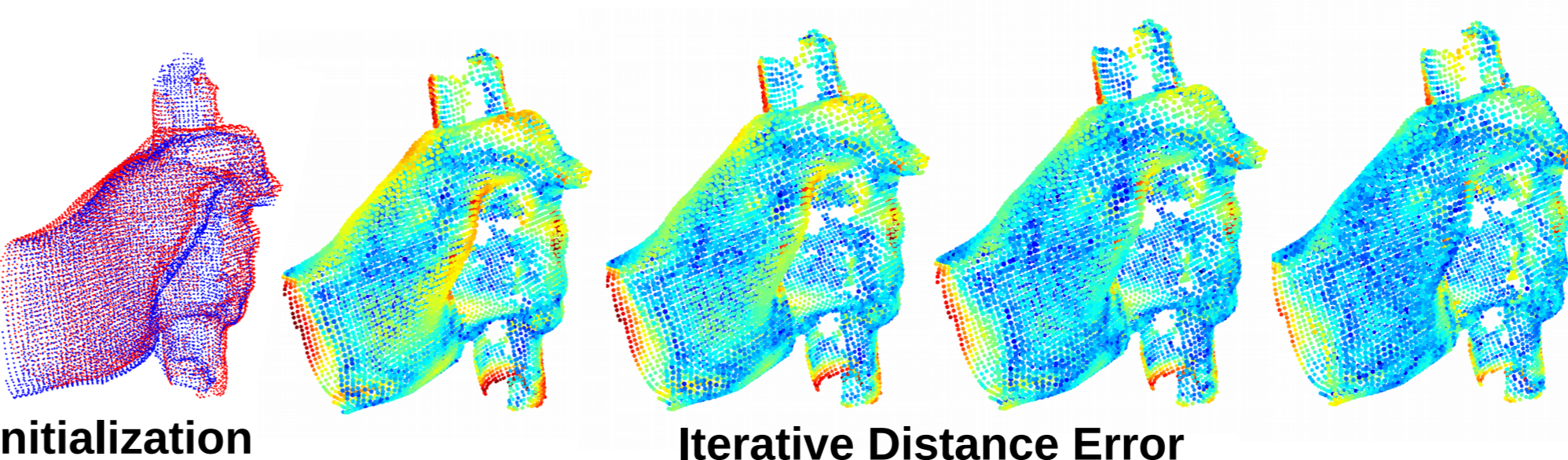
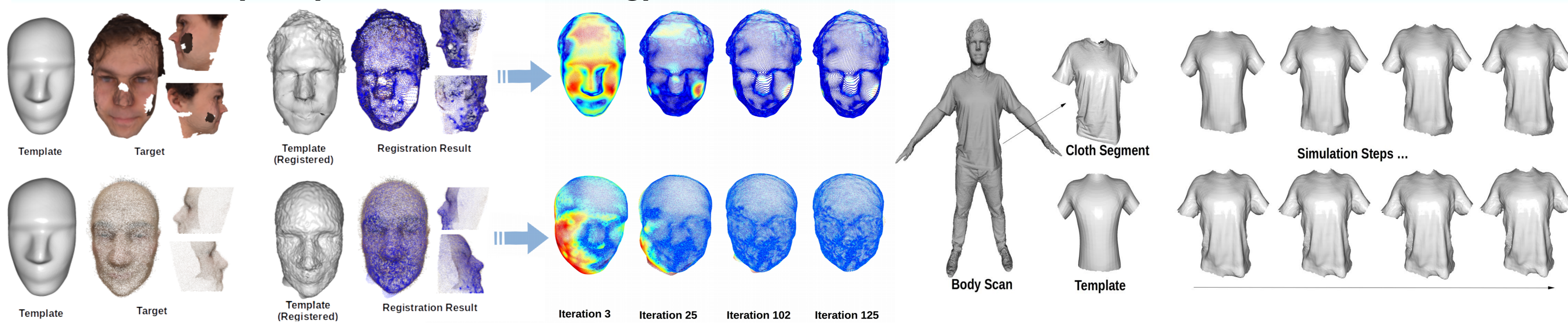
Image Registration



Performance Evaluation



Real Dataset (Template → Scan Fitting)



Acknowledgement

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